New integral operator for analytic functions

H. Özlem GÜNEY and Shigeyoshi OWA

Abstract

Let $A_p(n)$ be the class of functions $f(z)$ given by

$$f(z) = z^p + a_{p+n}z^{p+n} + a_{p+n+1}z^{p+n+1} + \dots$$

which are analytic in the open unit disc $\mathbb{U}$. For $f(z) \in A_p(n)$, new integral operators $\mathcal{O}_{-j}f(z)$ and $\mathcal{O}_j f(z)$ $(j = 0, 1, 2, ...)$ using some integral operators are considered. For such $\mathcal{O}_{-j}f(z)$ and $\mathcal{O}_j f(z)$, some interesting properties of $f(z)$ are discussed.

Keywords: Analytic function, integral operator, p-valently starlike of order α, p-valently convex of order α, dominant, subordination, m different boundary points.

2010 Mathematical Subject Classification: 30C45.

ON CERTAIN APPLICATIONS OF GRUNSKY COEFFICIENTS IN THE THEORY OF UNIVALENT FUNCTIONS

MILUTIN OBRADOVIĆ AND NIKOLA TUNESKI

Abstract. Let function f be normalized, analytic and univalent in the unit disk $\mathbb{D} = \{z : |z| < 1\}$ and $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. We denote by $\mathcal{S}$ the class of all such functions. Using a method based on Grusky coefficients we study several problems over the class $\mathcal{S}$: upper bound of the third logarithmic coefficient, upper bound of the coefficient difference $|a_4| - |a_3|$, upper bounds of the second and the third Hankel determinant, upper bounds of the second and the third Hankel determinant for inverse functions. Some of the obtained results improve the previous ones.

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1991 *Mathematics Subject Classification.* 30C45, 30C50, 30C55.
Key words and phrases. univalent functions, Grunsky coefficients, third logarithmic coefficient, coefficient difference, second Hankel determinant, third Hankel determinant.

The Progress of Fekete-Szegö Problems Related to Various Subclasses of Analytic Functions

Maslina Darus

Abstract

This article discusses on the progress of Fekete-Szegö problems for certain subclasses of analytic functions. The most important results recorded dated in 1933 by Fekete and Szegö [1] by giving sharp results for the functional $|a_3 - \mu a_2^2|$of a Taylor series. That was for the class analytic univalent functions S. Later, many tried to study for the subclasses of S, such as for the classes of starlike, convex and close-to-convex (see for examples:[2, 3, 4, 5, 6]. Throughout the decades, generalisation of subclasses of S began and many new results related to Fekete-Szegö were solved. These include the ones with differential operators, fractional calculus and the bi-univalent class of functions. Some earlier works and new results will be presented.

2010 Mathematics Subject Classification: 30C45, 30C50.
Key words and phrases: Fekete-Szegö problems, analytic functions.

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(p, q)–derivative on univalent functions associated with subordination structure

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October 2, 2021

Abstract

By means of Jackson's (p, q)–derivative a new class of univalent functions based on subordination is defined. We evoke some geometric properties such as coefficient estimate, convolution preserving, convexity and radii properties of this class of functions are obtained.

Keywords: Univalent function, Coefficient bounds, Convolution, Subordination, Convexity, Radii of starlikeness and convexity.

2010 Mathematics Subject Classification: 30C45, 30C50.

Nephroid starlikeness using hypergeometric functions

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Abstract. Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$ and $\mathcal{A}$ consist of analytic functions $f : \mathbb{D} \to \mathbb{C}$ satisfying the normalization conditions $f(0) = 0$ and $f'(0) = 1$. Recently, the authors in [1, 2] introduced the Ma-Minda type function family

$$\mathcal{S}^*_{Ne} := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi_{Ne}(z) = 1 + z - \frac{z^3}{3} \right\}$$

associated with a 2-cusped kidney-shaped curve called *nephroid* given by

$$\left((u-1)^2 + v^2 - \frac{4}{9} \right)^3 - \frac{4v^2}{3} = 0.$$

The authors in [1, 2] discussed in detail several geometrical and analytical properties of the family $\mathcal{S}^*_{Ne}$.

In this talk, we adopt a novel technique that uses the starlikeness properties of the *hypergeometric functions* (Gaussian and Kummer) to determine sharp estimates on β so that each of the differential subordinations

$$p(z) + \beta z p'(z) \prec \begin{cases} \sqrt{1+z}; \\ 1+z; \\ e^z; \end{cases}$$

imply $p(z) \prec \varphi_{Ne}(z) := 1 + z - \frac{z^3}{3}$, where $p(z)$ is analytic satisfying $p(0) = 1$. As applications, we establish conditions that are sufficient to deduce that $f \in \mathcal{A}$ is nephroid starlike in $\mathbb{D}$, i.e., $f \in \mathcal{S}^*_{Ne}$.

Keywords. Differential subordination, Starlike function, Lemniscate of Bernoulli, Cardioid, Nephroid

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The Fekete-Szego Theorem for Close-to-convex Functions Associated with The Koebe Type Function [1]

Sidik Rathi, Shaharuddin Cik Soh

Abstract

This paper deals with the class S containing functions which are analytic and univalent in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$. Functions f in S are normalized by $f(0) = 0$ and $f'(0) = 1$ and has the Taylor series expansion of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. In this paper we investigate on the subclass of S of close-to-convex functions denoted as $C_{g_\alpha}(\lambda, \delta)$ where function $f \in C_{g_\alpha}(\lambda, \delta)$ satisfies $\mathrm{Re}\{\ e^{i\lambda}\frac{zf'(z)}{g_\alpha(z)}\}$ for $|\lambda| < \frac{\pi}{2}$, $\cos(\lambda) > \delta$, $0 \le \delta < 1,\ 0 \le \alpha \le 1$ and $g_\alpha = \frac{z}{(1-\alpha z)^2}$. The aim of the present paper is to find the upper bound of the Fekete-Szego functional $|a_3 - \mu a_2{}^2|$ for the class $C_{g_\alpha}(\lambda, \delta)$. The results obtained in this paper is significant in the sense that it can be used in future research in this field, particularly in solving coefficient inequalities such as the Hankel determinant problems and also the Fekete-Szego problems for other subclasses of univalent functions.

2010 Mathematics Subject Classification: 30C45, 30C50
Key words and phrases: Univalent functions, Coefficient inequality problems, Fekete-szego problems, Close-to-convex function, Koebe function

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On the Upper Bound of the Third Hankel Determinant for Certain Class of Analytic Functions Related with Exponential Function.

Luminita COTIRLA

In the present paper we introduce a new class of analytic functions f in the open unit disk normalized by $f(0) = f'(0)-1 = 0$, associated with exponential functions. The aim of the present paper is to investigate the third-order Hankel determinant H_3(1) for this function class and obtain the upper bound of the determinant H_3(1).

On Certain Subclass of Starlike Functions with Negative Coefficients Associated with Erdelyi-Kober Integral Operator

Thomas Rosy, S.Prathiba

Abstract

In this research article, making use of Erdelyi-Kober integral operator, we define a new subclass $\mathcal{T}_{\mu}^{a,c}(\alpha, \beta, \gamma, A, B)$ of starlike functions with negative coefficient.Various properties like coefficient estimates, neighbourhood results, integral means, partial sums and subordination results are examined for this class.

2010 Mathematics Subject Classification: 30C45.
Key words and phrases: Univalent, starlike and convex functions, Erdelyi-Kober Integral operator.

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Hankel Determinant $H_2(3)$ for Certain Subclasses of Univalent Functions

Andy Liew Pik Hern, Aini Janteng, Rashidah Omar

Abstract

Let S to be the class of functions which are analytic, normalized and univalent in the unit disk $U = \{z : |z| < 1\}$. The main subclasses of S are starlike functions, convex functions, close-to-convex functions, quasi convex functions, starlike functions with respect to (w.r.t) symmetric points and convex functions w.r.t symmetric points which are denoted by $S^*, K, C, C^*, S_s^*, K_s$ respectively. In recent past, a lot of mathematicians studied about Hankel determinant for numerous classes of functions contained in S. The qth Hankel determinant for $q \geq 1$ and $n \geq 0$ is defined by $H_q(n)$. $H_2(1) = a_3 - a_2^2$ is greatly familiar so called Fekete-Szegö functional. It has been discussed since 1930's. Mathematicians still have lots of interest to this, especially in an altered version of $a_3 - \mu a_2^2$. Indeed, there are many papers explore the determinant $H_2(2)$ and $H_3(1)$. From the explicit form of the functional $H_3(1)$, it holds $H_2(k)$ provided k from 1-3. Exceptionally, one of the determinant that is $H_2(3) = a_3a_5 - a_4^2$. From this determinant, it consists of coefficients of function f which belong to the classes S_s^* and K_s so we may find the bounds of $|H_2(3)|$ for these classes. Likewise, we got the sharp results for S_s^* and K_s for which $a_2 = 0$ are obtained.

2010 Mathematics Subject Classification: 30C45, 33C20, 30C85.
Key words and phrases: Univalent Functions, Starlike Functions w.r.t Symmetric Points, Convex Functions w.r.t Symmetric Points, Hankel Determinant.

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Roper-Suffridge extension operators and Janowski univalent functions

Andra Manu

Abstract

In this paper, we will present certain properties that are satisfied on the unit ball $\mathbf{B}^n$ by the following Roper-Suffridge extension operators:

$$\Phi_{n,\alpha,\beta}(f)(z) = \Big(f(z_1), \tilde{z}\Big(\frac{f(z_1)}{z_1}\Big)^{\alpha}(f'(z_1))^{\beta}\Big),\ z = (z_1, \tilde{z}) \in \mathbf{B}^n,$$

where $\alpha, \beta \geq 0$, and

$$\Phi_{n,Q}(f)(z) = (f(z_1) + f'(z_1)Q(\tilde{z}), \tilde{z}\sqrt{f'(z_1)}),\ z = (z_1, \tilde{z}) \in \mathbf{B}^n,$$

where $Q : \mathbb{C}^{n-1} \to C$ is a homogeneous polynomial of degree 2. We will show that the above mentioned extension operators preserve the g-parametric representation, where the function g is given by $g(\zeta) = \frac{1+A\zeta}{1+B\zeta}$, $\zeta \in U$ and $-1 \leq B < A \leq 1$. Also, these extension operators preserve the Janowski starlikeness and the Janowski almost starlikeness.

Other particular cases will also be mentioned.

2010 Mathematics Subject Classification: 32H99, 30C45.
Key words and phrases: g-Loewner chain, g-parametric representation, Janowski starlikeness, Janowski almost starlikeness.

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Truncation on Infinitely small elements

Osman Hamza

Abstract

In this study, firstly we investigate the definition of truncation and its basic properties. After that we give infinitely small elements and truncated Riesz spaces and their relation between vector lattices and truncation.

2010 Mathematics Subject Classification: 46B40, 46B42
Key words and phrases: Truncation, Infinitely small, Truncated Riesz space

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Logarithmic coefficients bounds for the inverse of univalent functions

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(A joint work with S. Ponnusamy and K.-J. Wirths)

Abstract: Let $\mathcal{S}$ be the class of analytic and univalent functions in the unit disk $|z| < 1$, that have a series of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Let F be the inverse of the function $f \in \mathcal{S}$ with the series expansion

$$F(w) = f^{-1}(w) = w + \sum_{n=2}^{\infty} A_n w^n \quad \text{for } |w| < 1/4.$$

The logarithmic inverse coefficients Γ_n of F are defined by the formula

$$\log\left(\frac{F(w)}{w}\right) = 2\sum_{n=1}^{\infty} \Gamma_n(F) w^n.$$

In this talk, we will discuss the sharp bound for $|\Gamma_n(F)|$ when f belongs to $\mathcal{S}$ for all $n \geq 1$. This result motivates us to carry forward similar problems for some of its important geometric subclasses. In some cases, we have managed to solve this question completely but in some other cases it is difficult to handle for $n \geq 4$. For example, in the case of convex functions f, we investigated the logarithmic inverse coefficients $\Gamma_n(F)$ of F satisfy the inequality

$$|\Gamma_n(F)| \leq \frac{1}{2n} \text{ for } n \geq 1, 2, 3$$

and the estimates are sharp for the function $l(z) = z/(1-z)$. Although this cannot be true for $n \geq 10$, it is not clear whether this inequality could still be true for $4 \leq n \leq 9$.

keywords: Univalent function, Inverse function, starlike and convex functions, subordination, Inverse Logarithmic coefficients, Schwarz's lemma

This talk is based on the following article.

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Logarithmic coefficients bounds and coefficient conjectures for subclasses of univalent functions

Teodor Bulboacă, Ebrahim Analouei Adegani

Abstract

It is well-known that the logarithmic coefficients play an important role in development of the theory of univalent functions. If $\mathcal{S}$ denotes the class of functions $f(z) = z + \sum\limits_{n=2}^{\infty} a_n z^n$ analytic and univalent in the open unit disk $\mathbb{U}$, then the logarithmic coefficients $\gamma_n(f)$ of the function $f \in \mathcal{S}$ are defined by $\log \dfrac{f(z)}{z} = 2 \sum\limits_{n=1}^{\infty} \gamma_n(f) z^n$.

Based on some recent works we will discuss a few coefficient bounds conjectures and some partial solutions for different subclasses of univalent functions. The proofs of the main results involve an efficient method of Prokhorov and Szynal, Briot-Bouquet differential subordinations, etc.

We mention that several researchers have subsequently investigated similar problems regarding the logarithmic coefficients and the coefficient problems like Analouei Adegani, Ali, Vasudevarao, Alimohammadi, Cho, Ebadian, Kargar, Kumar, Obradović, Ponnusamy, etc., to mention a few of them.

2010 Mathematics Subject Classification: 30C45, 30C50, 30C80.
Key words and phrases: Univalent functions, starlike and convex functions of some order, Faber polynomial, subordination, logarithmic coefficients, close-to-convex functions.

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A new subclass of analytic functions connected with Mittag-Leffler-type Poisson distribution series

B. Venkateswarlu[1], P.Thirupathi Reddy[2] and Shashikala A[3]

Abstract

The object of this paper is to study the geometric properties such as the coefficient bounds , radii of close-to- convex and starlikeness and convex linear combinations for the class $TS^{m}_{\alpha,\beta}(\mu, \gamma, \varsigma)$. Furthermore, we obtained integral means inequalities for the functions of the defined class.

2010 Mathematics Subject Classification: 30C45, 30C50.
Key words and phrases: analytic, starlike,convex, integral means inequality, Mittag-Leffler function, Poisson distribution series.

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On Some Properties of a New Integral Operator

Nguyen Van Tuan, Roberta Bucur, Daniel Breaz

Abstract

For analytic functions in the open unit disk U, a new integral operator is introduced. The main objective of this paper is to obtain univalence for the given integral operator. Our main results contain some interesting corollaries as special cases.

2010 Mathematics Subject Classification: 30C45.
Key words and phrases: analytic, univalent, integral operator.

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EXTREMAL PROBLEMS FOR UNIVALENT FUNCTIONS OF ONE AND SEVERAL COMPLEX VARIABLES

Cristea (Deaconu) Daria-Roxana

Abstract

The paper entitled "Extremal Problems for Univalent Functions of One and Several Complex Variables" studies problems about extreme and support points of various subsets of $\mathcal{H}(U)$, where U is the unit disc in $\mathbb{C}$. We will investigate some problems about support points for different subsets of $\mathcal{H}(B^n)$, where B^n is the Euclidean open unit ball in $\mathbb{C}^n$. This paper is structured in four parts.

The first part, "Introductory notions", contains basic notions related to holomorphy in the complex plane.

The second part, "Univalent functions on the unit disc in $\mathbb{C}$", presents four families of normalized univalent functions on U which have different (geometrically) properties. Moreover, this part also presents the class of holomorphic functions which have positive real part.

The part entitled "Extremal problems for univalent functions" studies significant problems concerning the extreme and support points for the classes of functions introduced in the previous part. The theory of Loewner chain is very useful in our study.

The last part, "Univalent mappings on B^n", contains general results about univalent mapping on B^n ($n \in \mathbb{N}$, $n \geq 2$). Also, it presents fundamental differences between the one dimensional case and the n-dimensional case in the study of univalent mappings. Moreover, this part presents a modern method due to F. Bracci which allows us to deduce that there exists bounded support points of a special family of normalized univalent mapping on B^2.

The last part of this paper contains further research directions as the study of extremal problems of compact subsets $S(B^n)$. Furthermore, this paper presents significant open problems as the general structure of the sets ex$S$, supp$S$, ex$S^*(B^n)$, supp$S^*(B^n)$.

2010 Mathematics Subject Classification: 30Cxx.

Key words and phrases: Univalent functions, Starlike functions, Convex functions, Compact families, Extremal problems, Loewner chains.

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Bohr Radius for Goodman-Ronning Type Harmonic Univalent Functions

S.Sunil Varma and Thomas Rosy

Abstract

Let $\mathcal{H}$ denote the class of harmonic univalent functions $f = h + \overline{g}$ defined on the unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ where h and g are analytic functions in Δ with Taylor's series expansion $h(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = \sum_{n=1}^{\infty} b_n z^n$ about the origin with $|b_1| < 1$. Denote by $G_{\mathcal{H}}(\gamma)$ the subclass of Goodman-Ronning type harmonic univalent mappings introduced and studied in [3]. Let $G^0_{\mathcal{H}}(\gamma)$ be the subclass of $G_{\mathcal{H}}(\gamma)$ consisting of functions $f = h+\overline{g}$ where $h(z) = z - \sum_{n=2}^{\infty} |a_n| z^n$ and $g(z) = \sum_{n=2}^{\infty} |b_n| z^n$. In this paper we obtain the sharp Bohr radius, Bohr-Rogonoski radius, improved Bohr-radius and refined Bohr radius for the functions in the class $G^0_{\mathcal{H}}(\gamma)$.

2010 Mathematics Subject Classification: 30C45
Key words and phrases:Harmonic mappings, univalent functions, Goodman-Ronning type functions, Bohr radius, Bohr-Rogonoski radius

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On Brannan and Clunie's Conjecture for domains bounded by Conic sections involving $q-$difference operators

S. Kavitha

Abstract

In the present investigation, the author obtain initial coefficient bounds for the bi-close-to-convex functions in the function class Σ of bi-univalent functions defined in the open unit disk, which are associated with $q-$difference operator related to conic sections. We also obtain the Fekete-Szegö coefficient inequalities for the class of functions defined in this article. We also verify Brannan and Clunie's conjecture $|a_2| \leq \sqrt{2}$ for our classes.

2010 Mathematics Subject Classification: 30C45, 30C80 .
Key words and phrases: Brannan-Clunie conjecture, bi-univalent, bi-close-to-convex, q-difference operator, Taylor-Maclaurin series, coefficient estimate, Fekete-Szegö inequality.

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Classes with Negative Coefficient Involving q-Derivative Operator

Andy Liew Pik Hern, Aini Janteng, Rashidah Omar

Abstract

In this paper, we introduce classes with negative coefficient involving q-derivative which are q-starlike and q-convex, denoted by $S_q^*T(\alpha,\beta)$ and $K_qT(\alpha,\beta)$ of function f which are analytic and univalent in the open unit disk $D=\{z : |z|<1\}$ given by $f(z)=z-\sum_{n=2}^{\infty}a_nz^n$, $z\in D$. The coefficient estimates and growth results are obtained for these classes.

2010 Mathematics Subject Classification: Primary 30C45.
Key words and phrases: Negative Coefficient, q-Derivative Operator

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Analytic Classes with Probability distribution

The purpose of the present paper is to introduce a generalized discrete probability distribution in order to develop its connections with the normalized analytic subclasses whose coefficients are probabilities of the discrete probability distribution. We will explore some applications of this distribution with respect to the univalent functions. Moreover, we will derive different properties of these analytic classes such as coefficient bounds and integral preserving properties by using the techniques of convolution and subordination.

Toeplitz Determinants for a Subclass of Analytic Functions Involving q-Derivative Operator

Part Leam Loh, Aini Janteng, See Keong Lee

Abstract

Let A to be the class of analytic functions in the open unit disk $D = \{z \in \mathbb{C} : |z| < 1\}$ with $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. The class K_q is a subclass of A involving q-derivative operator. The paper investigates a study of finding estimates for coefficient inequalities and Toeplitz determinants whose elements are the coefficients a_n for $f \in K_q$.

2010 Mathematics Subject Classification: 05A30, 30C50, 15B05.
Key words and phrases: quantum (or q-) calculus, q-derivative operator, analytic functions, Toeplitz determinant.

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New Subclasses of Bi-Univalent Functions based on the Fibonacci Numbers

Munirah Rossdy, Rashidah Omar, Shaharuddin Cik Soh

Abstract

In this work, by using the Al-Oboudi differential operator and the rule of subordination, we introduced the new subclasses $D^{n,\rho}_{\Sigma,\delta}(\Phi)$ and $F^{n,\alpha}_{\Sigma,\delta}(\Phi)$ of the bi-univalent functions. Likewise, we use the Fibonacci numbers to derive the initial coefficients bounds for $|a_2|$ and $|a_3|$ of the bi-univalent function subclasses.

2010 Mathematics Subject Classification: xxxxx, xxxxx.
Key words and phrases: Al-Oboudi differential operator, fibonacci numbers, subordination and bi-univalent functions.

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On special differential subordinations using fractional integral of Sălăgean and Ruscheweyh operators

Alina Alb Lupaş

Abstract

In the present paper a new operator denoted by $D_z^{-\lambda} L_\alpha^n$ is defined using the fractional integral of Sălăgean and Ruscheweyh operators. By means of the newly obtained operator, a new subclass of analytic functions in the unit disc denoted by $S_n(\delta, \alpha, \lambda)$ is introduced and various properties and characteristics of this class are derived making use of the concept of differential subordination. Also, several interesting differential subordinations are established regarding the operator $D_z^{-\lambda} L_\alpha^n$.

2010 Mathematics Subject Classification: 30C45, 30A20, 34A40.
Key words and phrases: differential subordination, convex function, best dominant, differential operator, fractional integral.

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On Certain Classes of Analytic functions of Complex order defined by Erdelyi-Kober Integral operator

Thomas Rosy, Asha Thomas

Abstract

In this paper, we consider new subclasses $\mathfrak{TS}_n(\mu, \mathfrak{a}, \mathfrak{b}, \ell, \tau, \gamma)$ and $\mathfrak{TR}_n(\mu, \mathfrak{a}, \mathfrak{b}, \ell, \tau, \gamma)$ of analytic univalent functions defined by Erdelyi-Kober integral operator. We obtain coefficient inequalities, inclusion relationships involving the (n, δ)-neighborhoods, partial sums and integral mean inequalities for the functions that belongs to these classes. Also, subordinating factor sequence for the functions in the classes $\mathfrak{S}_n(\mu, \mathfrak{a}, \mathfrak{b}, \ell, \tau, \gamma)$ and $\mathfrak{R}_n(\mu, \mathfrak{a}, \mathfrak{b}, \ell, \tau, \gamma)$ are derived.

2010 Mathematics Subject Classification: Primary 30C45.
Key words and phrases: univalent functions, generalised hypergeometric functions,Hadamard product, integral means, subordinating factor sequence.

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Remarks on Some Convex Combinations of Graham-Kohr Extension Operators

Eduard Ştefan Grigoriciuc

Abstract

Starting from a result proved by P.N. Chichra and R. Singh [1, Theorem 2] which says that if a function f is starlike with the property that $\mathrm{Re}[f'(z)] > 0$, then $(1 - \lambda)z + \lambda f(z)$ is also starlike on the unit disc U, for all $\lambda \in (0, 1)$, we discuss in this paper about convex combinations of biholmorphic mappings on the Euclidean unit ball in the case of several complex variables. Moreover, we consider not only biholomorphic mappings, but also convex combinations of extension operators. The main extension operator that will be considered in this paper is the extension operator defined by I. Graham and G. Kohr in [3].

2010 Mathematics Subject Classification: 32H02, 30C45.
Key words and phrases: Biholomorphic mapping, Starlike mapping, Convex combination, Extension operator, Loewner chain.

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On a subclass of close to convex functions

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Abstract

Let $\mathcal{H}$ be the class of all holomorphic functions in the open unit disc $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, and $\mathcal{A}$ the subclass of $\mathcal{H}$ of functions $h \in \mathcal{H}$ with the normalisation $h(0) = h'(0) - 1 = 0$. Thus functions $h \in \mathcal{A}$ has representation

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbb{D}.$$

Denote $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of univalent functions.

In this talk, we define and investigate a subclass close-to-convex functions evolving from Robertson's analytic condition for starlike functions with respect to a boundary point, combined with subordination. Examples of some new subclasses are presented. Initial coefficient estimates are given and a Fekete-Szegö inequality is obtained . Differential subordinations involving these newly defined subclasses are also established.

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Some geometric aspects of non-linear resolvents (Dedicated to the memory of Professor Gabriela Kohr)

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Let f belongs to the set of all infinitesimal generators of one-parameter semigroups of holomorphic self-mappings on the open unit disk vanishing at zero. Denote $\mathcal{J} = \left\{(I + rf)^{-1},\ r > 0\right\}$, the family of resolvents of such generators. The aim of my talk is to present properties of this family in the spirit of geometric function theory obtained in [1–2].

We discovered, in particular, that resolvents form an inverse Löwner chain of hyperbolically convex functions. Moreover, every resolvent is a starlike function of order that grows from $\frac{1}{2}$ to 1. In turn, this implies that the family of normalized resolvents converges to the identity map. These results follow from distortion and covering theorems for resolvents we establish. Also, any resolvent admits quasiconformal extension to the complex plane $\mathbb{C}$. We prove that any element of $\mathcal{J}$ is also a generator and obtain some characteristics of semigroups generated by them. The existence/non-exoistence of repelling fixed points of resolvents is also studied.

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